MATHEMATICAL MODELLING OF MODIFIED GAIN DOPPLER BEARING EXTENDED KALMAN FILTER FOR UNDERWATER TRACKING

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Abstract—The paper concerns the estimation of Target Motion Parameters (TMP) viz. Range, Bearing, Course and speed of a moving target, from noisy corrupted data. A sensor is used to obtain the Frequency and Bearing parameters of the target in this type of environment. The estimate of all TMPs is obtained by processing of these parameters data.

Doppler Bearing passive target tracking is the determination of the trajectory from measurements (bearing and doppler) of signals from the target. In the underwater scenario, the passive SONAR detects the sound waves in water using Hydrophone sensors. Kalman filter is used to compute target motion parameters (TMP) from the a priori Bearing and Frequency information and is mainly applied in linear systems. Extended Kalman Filter (EKF) is the most widely applied state estimation algorithm for nonlinear systems. The algorithm has been implemented in MATLAB, a language of technical computing, widely used in research, engineering and scientific computations for mathematical modelling of Modified Gain Extended Kalman Filter.

Keywords- TMA, Doppler Bearing; kalman filter; Underwater tracking; passive sonar; modified gain extended kalman filter; Mathematical Modelling

Introduction

Target motion analysis (TMA) is a process by which targets around a naval vessel are identified and tracked. This is only an approximation of the target location based on low signal-to-noise data. Methods have been documented for reducing the cognitive load on an operator [1].

TMA provides a means to track targets around using bearing data received from passive sonar in deciding an appropriate course of action (Streit and Walsh)[2]. refers to the current observation platform. A bearing is essentially an indication of direction with respect to the observation platform. Target (aka source) refers to a single vessel being tracked by . The problem space is the set all possible scenarios of the location and movement of a target confirmed by the bearings. A solution is considered to be single resolution to the location and movement of a target based in the problem space.

Target motion analysis (TMA) using conventional passive bearing together with frequency measurements was studied. This approach offered tactical advantage over the classical bearings-only TMA. It makes the ownship maneuver superfluous. The inclusion of range, course, and speed parameterization is proposed in the UKF target state vector to obtain the convergence of the solution fast (S.Koteswararao)[3].

Own ship monitors noisy sonar bearings from a target, which is assumed to be travelling with a uniform velocity. The own ship processes these measurements and finds out target motion parameters, namely, range, course, bearing, and speed of the target. Here the measurement is nonlinear, making the whole process nonlinear. Added to this, since bearing measurements are extracted from a single sensor, the process remains unobservable until own ship executes a proper maneuver. However, there are many methods available [4-9] to obtain target motion parameters in the above situation.

The extended Kalman filter applied to bearings-only target tracking is theoretically analyzed. Closed-form expressions for the state vector and its associated covariance matrix are introduced, and subsequently used to demonstrate how bearing and range estimation errors can interact to cause filter instability (i.e., premature covariance collapse and divergence). Further investigation reveals that conventional initialization techniques often precipitate such anomalous behavior. These results have important practical implications and are not presently being exploited to full advantage. In particular, they suggest that substantial improvements in filter stability can be realized by employing alternative initialization and relinearization procedures. Some candidate methods are proposed and discussed [10].

2.0 Kalman Filtering

Ever since the manual tracing of "blips" on radar and sonar systems evolved into computer controlled tracking algorithms, tracking of any kind of sensor data continued to develop significantly. Tracking is “the processing of
measurements obtained from a target in order to maintain an estimate of its current state” [11].

The Kalman filter algorithm has proven to be quite successful in filtering sonar data [12]. It is an estimation algorithm that considers the current state, the control input, noisy observations and provides the optimal linear estimate of its state along with the associated error variance. Kalman Filtering attracted considerable attention because of general validity, mathematical elegance and widespread technical application [13]. Another striking feature of the Kalman Filter is the number of different ways, maximum likelihood method, minimum variance method and the least-squares method, a solution equation can be derived. [14].

The feasibility of Song and Speyer’s modified gain extended Kalman filter using bearings-only measurements is explored for underwater applications. A much simpler version of the modified function introduced by Galkowski and Islam is considered for the development of this algorithm. This algorithm estimates target motion parameters and detects target manoeuvre, using zero mean chi-square distributed random sequence residuals, in sliding window format. During the period of target manoeuvring, the covariance of the process noise is increased sufficiently in such a way that the disturbance in the solution is less. When a target manoeuvre is completed, the covariance of process noise is lowered. The performance of this algorithm is evaluated in Monte Carlo simulation and results are shown for various typical geometries [15].

Kalman filter is an efficient recursive filter that estimates the state of a linear dynamic system from a series of noisy measurements. Kalman Filter utilizes a set of mathematical equations that implement a predictor-corrector type estimator that is optimal in the sense that it minimizes the estimated error covariance — when some presumed conditions are met. With the initial assumed estimates it allows the model parameters to be predicted and adjusted with each new measurement, providing an estimate of error at each update. Its ability to incorporate the effects of noise, and its computation structure, has made it popular for use in computer vision tracking applications [16].

Figure 1: Complete picture of the operation of Kalman Filter, combining the high-level diagram with the equations.

### 2.1. Mathematical modeling of MGDBEKF: Target State and Measurement Equations:

The target, located at the coordinates (x, y), moves with a constant velocity. Let the target state vector be \( X(k) \), where

\[
X_k(k) = \begin{bmatrix}
\hat{x}_1(k) \\
\hat{y}_1(k) \\
r_x(k) \\
r_y(k) \\
F_s(k)
\end{bmatrix}
\]

Where \( \hat{x}(k) \) and \( \hat{y}(k) \) are target velocity components in X and Y direction, and \( r_x(k) \) and \( r_y(k) \) are range components in X and Y direction respectively.

The observer state is similarly defined as

\[
X_o(k) = \begin{bmatrix}
\hat{x}_o(k) \\
\hat{y}_o(k) \\
x_o(k) \\
y_o(k)
\end{bmatrix}
\]

It is assumed that the target is moving with constant course and velocity, then,

\[
\dot{x}_1(k+1) = x_1(k) \\
\dot{y}_1(k+1) = y_1(k) \\
r_x(k+1) = r_x(k) + x_1(k).t_s - (x_o(k+1) - x_o(k)) \\
r_y(k+1) = r_y(k) + y_1(k).t_s - (y_o(k+1) - y_o(k)) \\
F_s(k+1) = F_s(k)
\]

These are called as prediction equations.

Arranging the above equations in matrix form,

\[
\begin{bmatrix}
\dot{x}(k+1) \\
\dot{y}(k+1) \\
r_x(k+1) \\
r_y(k+1)
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x(k) \\
y(k) \\
x_o(k) \\
y_o(k)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
x_o(k+1) - x_o(k) \\
y_o(k+1) - y_o(k)
\end{bmatrix}
\]

Assuming that the target is non-maneuvering, the target state dynamic equation is given by

\[
X(k+1) = \phi(k+1/k)X(k) + b(k+1)
\]

Where \( \phi(k+1/k) \) and \( b(k+1) \) are transient matrix and the deterministic vector respectively. They are given by

\[
\phi(k+1/k) = \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
b(k+1) = \begin{bmatrix}
0 \\
0 \\
x_o(k+1) - x_o(k) \\
y_o(k+1) - y_o(k)
\end{bmatrix}
\]
The mathematical model of Modified Gain Doppler Bearing Extended Kalman Filter (MGDBEKF) is as follows,

\[ \frac{\partial B}{\partial x} = 0; \quad \frac{\partial B}{\partial y} = 0; \quad \frac{\partial B}{\partial \phi} = 0; \]

\[ \frac{\partial B}{\partial r_x} = \frac{1}{r^2} \left( \frac{x}{r} \right) \quad \frac{1}{r^2} \left( \frac{y}{r} \right) = \frac{\cos B}{r}; \]

\[ \frac{\partial B}{\partial r_y} = \frac{1}{r^2} \left( -\frac{x}{r} \right) = -\frac{\sin B}{r}; \]

\[ \frac{\partial F_m}{\partial x} = -\frac{\dot{F}_s(k) \sin \hat{B}(k)}{C}; \]

\[ \frac{\partial F_m}{\partial y} = -\frac{\dot{F}_s(k) \cos \hat{B}(k)}{C}; \]

\[ \frac{\partial F_m}{\partial \tau_x} = 0; \quad \frac{\partial F_m}{\partial \tau_y} = 0; \]

\[ \frac{\partial F_m}{\partial F_s} = \left( 1 + \frac{\dot{x}_r(k) \sin \hat{B}(k) + \dot{y}_r(k) \cos \hat{B}(k)}{C} \right); \]

We have the measurement equation

\[ Z(k) = H(k)X_s(k) + \xi(k) \]

Where

\[ H(k) = \begin{bmatrix} \frac{\partial B}{\partial x} & \frac{\partial B}{\partial y} & \frac{\partial B}{\partial \phi} & \frac{\partial B}{\partial r_x} & \frac{\partial B}{\partial r_y} & \frac{\partial B}{\partial F_s} \\ \frac{\partial F_m}{\partial x} & \frac{\partial F_m}{\partial y} & \frac{\partial F_m}{\partial \tau_x} & \frac{\partial F_m}{\partial \tau_y} & \frac{\partial F_m}{\partial F_s} & \frac{\partial F_m}{\partial F_s} \end{bmatrix} \]

Where,

\[ \text{TERM} = \left( 1 + \frac{\dot{x}_r(k) \sin \hat{B}(k) + \dot{y}_r(k) \cos \hat{B}(k)}{C} \right) \]

2.1.1. Input Covariance Matrix:

The noise is assumed zero-mean white noise and the variance of the random noise is given by

\[ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \]

Where \( x_o \) and \( y_o \) are Observer position components respectively. True North convention is followed for all angles to reduce mathematical complexity and easy implementation.

The equations of frequency, F and of bearing, B are non-linear. So these non-linear equations are linearised using the first term of Taylor’s Series expansion.
Noise in bearing and frequency measurements are uncorrelated.

\[
\text{∴ The Input Covariance matrix is given by}
\begin{bmatrix}
\sigma_B^2 & 0 \\
0 & \sigma_D^2
\end{bmatrix}
\]

Where \(\sigma_B^2\) is the variance of noise in bearing measurement.

\(\sigma_D^2\) is the variance of noise in frequency measurement.

2.1.2 Calculation of Kalman gain:

Kalman gain = \(P \cdot H' \cdot \text{inv}((H' \cdot P' \cdot H') + \text{Incov})\);

Where covariance matrix, \(P\) is defined as

\[
P(0/0) = \begin{bmatrix}
term1 & 0 & 0 & 0 & 0 \\
0 & term2 & 0 & 0 & 0 \\
0 & 0 & term3 & 0 & 0 \\
0 & 0 & 0 & term4 & 0 \\
0 & 0 & 0 & 0 & term5
\end{bmatrix}
\]

Where \(\text{term1} = (1/3) \cdot X(1)^2;\)
\(\text{term2} = (1/3) \cdot X(2)^2;\)
\(\text{term3} = (1/3) \cdot X(3)^2;\)
\(\text{term4} = (1/3) \cdot X(4)^2;\)
\(\text{term5} = (1/3) \cdot X(5)^2;\)

2.1.3 Correction or Measurement updation:

Correcting state estimate, \(X = X + (\text{kalmangain} \cdot D)\);

Where Deviation, \(D = \begin{bmatrix} [Bm - Bpred] \\ [Fm - X(5)] \end{bmatrix}\)

\(\text{Bm and Fm are the measured bearing and frequency obtained (with additive noise) from the simulator.}\)

Correction of Covariance, \(P = (I - \text{kalmangain} \cdot g) \cdot P \cdot (I - \text{kalmangain} \cdot g)' + (\text{kalmangain} \cdot \text{Incov} \cdot \text{kalmangain}');\)

Where Modified kalman gain, \(g = \begin{bmatrix}
\partial Bm/\partial x & \partial Bm/\partial y & \partial Bm/\partial X & \partial Bm/\partial r_x & \partial Bm/\partial r_y & \partial Bm/\partial F_s \\
\partial Fm/\partial x & \partial Fm/\partial y & \partial Fm/\partial X & \partial Fm/\partial r_x & \partial Fm/\partial r_y & \partial Fm/\partial F_s
\end{bmatrix}\)

Doppler shift is defined as the ratio of relative velocity to wavelength. It is given by

\[
\text{Doppler shift} = \frac{\text{relative velocity}}{\text{ wavelength}} = \frac{C/f_s}{C}\
\]
So, the frequency measured as observer is given by

\[
f_m(k) = f_s(k) + \frac{\text{relative velocity}}{C} \cdot f_s(k) + \gamma_f(k)
\]

\[
f_s(k) + \frac{\text{relative velocity}}{C} + \gamma_f(k)
\]

2.2.0. Modified gain doppler bearing extended kalman filter (MGDBEKF):

Modified Gain Extended Kalman Filter is one type of extended kalman filter whose gain is a function of past measurements and in particular the nonlinearities are modifiable. The familiar modified gain extended Kalman filter equations are given by

2.2.1. Prediction:

State:

\[
X(k + 1/k) = \Phi(k + 1/k)X(k/k) + b(k + 1)
\]

Covariance:

\[
P(k + 1/k) = \Phi(k + 1/k)P(k/k)\Phi^T(k + 1/k) + Q(k + 1)
\]

2.2.2. Kalman Gain:

\[
g(K+1) = P(k + 1/k)H^T(k + 1)\left[/(k + 1)P(k + 1/k)H^T(k + 1) + R(k + 1)\right]\]^{-1}

\[
g(K+1) = \begin{bmatrix}
\frac{\partial Bm(K+1)}{\partial x} & \frac{\partial Bm(K+1)}{\partial y} & \frac{\partial Bm(K+1)}{\partial x} & \frac{\partial Bm(K+1)}{\partial y} & \frac{\partial Bm(K+1)}{\partial x} & \frac{\partial Bm(K+1)}{\partial y}
\end{bmatrix}
\]

2.2.3. Correction:

State:

\[
X(k + 1/k + 1) = X(k + 1/k) + G(K+1)\left[Z(k + 1) - \hat{Z}(k + 1)\right]
\]

2.2.4. Covariance:

\[
P(K+1/K+1) = [I - G(K+1)g(K+1)]P(K+1/K)[I - G(K+1)g(K+1)]^T + GRG^T
\]

Where \( Q(k+1) \) is covariance of the plant noise, \( \hat{Z}(k + 1) \) is the estimated measurement and \( g(k+1) \) is the modified kalman gain at \( K+1 \).

3.0. Results and Discussion

3.1. Scenario-1

Initial Velocity of the Target = 15.00 mts, Initial Velocity of the = 10.00 mts, Initial Target Course = 60.00 degrees, Initial Observer Course = 90.00 degrees, Initial Range = 1800 meters, Initial Bearing = 10.00 degrees, Initial Position of the observer= (0.0, 0.0), Sigma in bearing = 0.17 deg, Sigma in frequency= 0.33 Hz, Target Frequency = 800.00 Hz

Various plots of the Target and Observer’s Position, Range, Bearing, Course & Speed Error analysis plots are shown in Figure-2 & 3.

Duration of run : 180 sec, Convergence Obtained: Range is converged at 163rd sec, Bearing is converged at 13th sec, Course is converged at 118th sec, Speed is converged at 112th sec.

3.2. Scenario-2

Initial Velocity of the Target = 15.00 mts, Initial Velocity of the = 5.00 mts, Initial Target Course = 50.00 degrees, Initial Observer Course = 145.00 degrees, Initial Range = 6000 meters, Initial Bearing = 50.00 degrees, Initial Position of the observer= (0.0, 0.0), Sigma in bearing= 0.17 deg, Sigma in frequency= 0.33 Hz, Target Frequency = 800.00 Hz, Various plots of the Target and Observer’s Position, Course & Speed and Error analysis plots are shown in Figure- 4 & 5.
Fig-4: Plot of ownship versus target position (Actual and predicted)

Fig-5: Range, Bearing, Course and Speed error plots

Duration of run : 190 sec, Convergence Obtained: Range is converged at 9th sec, Course is converged at 30th sec, Speed is converged at 111th sec, Bearing is converged at 11th sec.

NOMENCLATURE:

\( \hat{X}(k+1/k) \) : State vector estimate for time \( t(k+1) \), based upon the state vector estimate at time \( t(k) \).

\( \hat{X}(k+1/k+1) \) : State vector estimate for time \( t(k+1) \), based upon the state vector estimate at time \( t(k+1) \).

\( P(k/k) \) : Estimate of Covariance Matrix of \( \hat{X}(k/k) \) for time \( t(k) \), including a measurement at time \( t(k) \).

\( P(k+1/k) \) : Estimate of Covariance Matrix of \( \hat{X}(k+1/k) \) for time \( t(k+1) \), including a measurement at time \( t(k) \).

\( P(k+1/k+1) \) : Estimate of Covariance Matrix of \( \hat{X}(k+1/k+1) \) for time \( t(k+1) \), including a measurement at time \( t(k+1) \).

\( B_m \) : Bearing measurement at time \( t(k+1) \).

\( R_m \) : Range measurement at time \( t(k+1) \).

\( B(k+1) \) : True Bearing at time \( t(k+1) \).

\( U_k \) - Known Input

A – Transition matrix

B - Deterministic vector

\[ Q = \begin{bmatrix}
    0 & 0.5 * ts^3 & 0 & 0 & 0 \\
    0 & ts^2 & 0 & 0.5 * ts^3 & 0 \\
    0 & 0.5 * ts^3 & 0 & 0.25 * ts^4 & 0 \\
    0 & 0 & 0.25 * ts^4 & 0 & 0 \\
    0.00001 & 0 & 0 & 0 & 1.00001
  \end{bmatrix} \]

CONCLUSION

In this paper, an approach using a Modified Gain Extended Kalman Filter is proposed to estimate target motion parameters without using ownship maneuver in passive target tracking. Modified Gain Extended Kalman Filter (which is useful for nonlinear applications), a highly computational estimator was then used to filter the noisy measurements and estimate the Target Motion Parameters. Monte-Carlo Simulation was carried out in the end in a number of scenarios. The Project was successfully completed, tested and documented. The entire algorithm has been implemented in MATLAB, a language of technical computing, widely used in research, engineering and scientific computations.

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